Propogation of e.m. waves in a linear Media:-

If the e.m. waves travels in a linear & Homogeneous medium (i.e. μ and ε do not vary from point to point) the maxwell's eqn for this case can be written as,

$$\left.\begin{aligned}
&(i)\quad \vec{\nabla}\cdot\vec{E}=0\\
&(ii)\quad \vec{\nabla}\cdot\vec{B}=0\\
&(iii)\quad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\
&(iv)\quad \vec{\nabla}\times\vec{B}=\mu\varepsilon\frac{\partial\vec{E}}{\partial t}
\end{aligned}\right\} \text{——— (1)}$$

e.m. waves propogates through this medium, at a speed,

$$v=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{c}{n}$$

where,

$$n=\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$ is the "refractive index."

(ε_0, μ_0 in free space)

Let us, discuss the propogation waves from one medium to another by applying boundry conditions (55)

boundry Condⁿ are,

Tangental Comp. (normal Com.) $D_1^{\perp} - D_2^{\perp} = \sigma_f$ (e.f.) (discontinuous)

$B_1^{\perp} = B_2^{\perp} = 0$ (continuous)

parallell Comp. $E_1^{\parallel} - E_2^{\parallel} = 0$ (Continuous)

$H_1^{\parallel} - H_2^{\parallel} = K_f \times \hat{n}$ (m.f.) (discontinuous)

σ - Charge density
k - Current densities

assuming, all are Continuous, then

$$\left.\begin{aligned} &(i) \quad \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} \\ &(ii) \quad B_1^{\perp} = B_2^{\perp} \\ &(iii) \quad E_1^{\parallel} = E_2^{\parallel} \\ &(iv) \quad \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} \end{aligned}\right\} \longrightarrow (2)$$

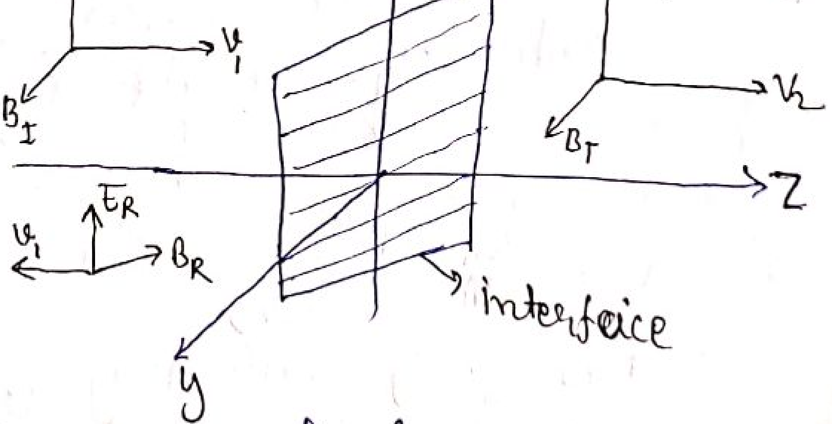
we use them, to deduce the laws governing reflection & refraction of e.m. waves

Here we will Consider two cases:-

Case I:-

V. Imp (for exam)

Reflection and transmission at normal incidence:



Let, a plane wave of frequency ω be travelling in z-direction and incident at the boundry of the medium from the

56 left. A part of the wave is reflected back to medium ①, another part is transmitted to the medium ②.

The electric & magnetic fields of incident wave are given by,

$$\left.\begin{aligned}\vec{E}_I(\vec{r},t) &= \tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{x}\\ \vec{B}_I(\vec{r},t) &= \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{y}\end{aligned}\right\} \text{——} ③$$

The electric & m.f. of Reflected wave are given by,

$$\left.\begin{aligned}\vec{E}_R(\vec{r},t) &= \tilde{E}_{0R}\, e^{i(k_1 z-\omega t)}\,\hat{x}\\ \vec{B}_R(\vec{r},t) &= -\frac{1}{v_1}\tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{y}\end{aligned}\right\} \text{——} ④$$

The e.f. & m.f of transmitted wave are given by,

$$\left.\begin{aligned}\vec{E}_T(\vec{r},t) &= \tilde{E}_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{x}\\ \vec{B}_T(\vec{r},t) &= \frac{1}{v_2}\tilde{E}_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{y}\end{aligned}\right\} \text{——} ⑤$$

Since, waves are normally incident, $B_1 = B_I + B_R$ for this case, the tangential Comp. of e.f & m.f. are Continuous. ~~at~~ the interface. only the parallel Comp. of e.f. & m.f. may be observed by applying the boundry Conditions of (iii) & (iv)

On Applying boundry Condⁿ (iii) — which is,

$$E_1^{\parallel} = E_2^{\parallel}$$

then,

$$\boxed{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}} \text{——} ⑥$$

on Applying boundary Condⁿ (iv) -

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{\vec{E}}_{0I} - \frac{1}{v_1}\tilde{\vec{E}}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{\vec{E}}_{0T}\right)$$

$$\frac{1}{\mu_1 v_1}\left(\tilde{\vec{E}}_{0I} - \tilde{\vec{E}}_{0R}\right) = \frac{1}{\mu_2 v_2}\left(\tilde{\vec{E}}_{0T}\right)$$

$$\tilde{\vec{E}}_{0I} - \tilde{\vec{E}}_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2}\tilde{\vec{E}}_{0T}$$

$$\boxed{\tilde{\vec{E}}_{0I} - \tilde{\vec{E}}_{0R} = \beta\,\tilde{\vec{E}}_{0T}} \quad \text{—— (7)}$$

where, $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$ $\left(\because \frac{v_1}{v_2} = \frac{n_2}{n_1}\right)$

Solving (6) & (7), we get

$$\tilde{\vec{E}}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{\vec{E}}_{0I}$$

$$\tilde{\vec{E}}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{\vec{E}}_{0I} \quad \text{—— (8)}$$

If permittivities μ are closed to their vaccum values i.e. $\underline{\mu_1 = \mu_2}$, then

$$\beta = \frac{v_1}{v_2} \qquad \frac{1-\frac{v_1}{v_2}}{1+\frac{v_1}{v_2}}$$

then eqⁿ (8) becomes,

$$\tilde{\vec{E}}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{\vec{E}}_{0I}$$

$$\& \ \tilde{\vec{E}}_{0T} = \left(\frac{2 v_2}{v_2 + v_1}\right)\tilde{\vec{E}}_{0I} \quad \text{—— (9)}$$

If, $\underline{v_2 > v_1}$ — then conclusion that the reflected waves is in phase with the incident waves.

If $v_1 > v_2$ —

The reflected wave is out of phase with the incident wave.

eq^n (9) can be expressed in term of refractive index 'n', as :—

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$$\left.\begin{aligned} E_{0R} &= \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_{0I} \\ \& \; E_{0T} &= \left(\frac{2n_1}{n_1 + n_2}\right) E_{0I} \end{aligned}\right\} \quad \text{——— (10)}$$

the intensity (Average power per unit Area) I is given by,

$$\boxed{I = \frac{1}{2} \epsilon v E_0^2} \qquad \epsilon v \frac{A_0^2}{2}$$

therefore, the ratio of reflective intensity to incident intensity becomes,

Ray 1

$$'R' = \frac{I_R}{I_I} = \frac{E_{0R}^2}{E_{0I}^2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad \text{——— (11)}$$

where 'R' is 'Reflection Coefficient'.

the Ratio of transmittance intensity to incident intensity, is given by:

$$'T' = \frac{I_T}{I_I}$$

wher 'T' is 'Transmittance Coefficient'.

Ray 1 & Ray 2

$$= \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \left(\frac{E_{0T}}{E_{0I}}\right)^2$$

$$'T' = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \quad \text{——— (12)}$$

R and T measure the fraction of the incident energy (from eqn (11) & (12).), that is reflected & transmitted as,

$$\boxed{R+T=1}$$

~~R and T measure the fraction~~ as Conservation of energy. of course, requires

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